The Behavior of Naturally Fractured Reservoirs

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ABSTRACT

An idealized model has been developed for the purpose of studying the characteristic behavior of a permeable medium which contains regions which contribute significantly to the pore volume of the system but contribute negligibly to the flow capacity; e.g., a naturally fractured or vugular reservoir. Unsteady-state flow in this model reservoir has been investigated analytically. The pressure build-up performance has been examined in some detail; and, a technique for analyzing the build-up data to evaluate the desired parameters has been suggested. The use of this approach in the interpretation of field data has been discussed.

As a result of this study, the following general conclusions can be drawn:

1. Two parameters are sufficient to characterize the deviation of the behavior of a medium with "double porosity" from that of a homogeneously porous medium.

2. These parameters can be evaluated by the proper analysis of pressure build-up data obtained from adequately designed tests.

3. Since the build-up curve associated with this type of porous system is similar to that obtained from a stratified reservoir, an unambiguous interpretation is not possible without additional information.

4. Differencing methods which utilize pressure data from the final stages of a build-up test should be used with extreme caution.

INTRODUCTION

In order to plan a sound exploitation program or a successful secondary-recovery project, sufficient reliable information concerning the nature of the reservoir-fluid system must be available. Since it is evident that an adequate description of the reservoir rock is necessary if this condition is to be fulfilled, the present investigation was undertaken for the purpose of improving the fluid-flow characterization, based on normally available data, of a particular porous medium.

DISCUSSION OF THE PROBLEM

For many years it was widely assumed that, for the purpose of making engineering studies, two param-

eters were sufficient to describe the single-phase flow properties of a producing formation, i.e., the absolute permeability and the effective porosity. It later became evident that the concept of directional permeability was of more than academic interest; consequently, the degree of permeability anisotropy and the orientation of the principal axes of permeability were accepted as basic parameters governing reservoir performance. [1,2] More recently, [3-6] it was recognized that at least one additional parameter was required to depict the behavior of a porous system containing regions which contributed significantly to the pore volume but contributed negligibly to the flow capacity. Microscopically, these regions could be "dead-end" or "storage" pores or, macroscopically, they could be discrete volumes of low-permeability matrix rock combined with natural fissures in a reservoir. It is obvious that some provision for the inclusion of all the indicated parameters, as well as their spatial variations, must be made if a truly useful, conceptual model of a reservoir is to be developed.

A dichotomy of the internal voids of reservoir rocks has been suggested.[7,8] These two classes of porosity can be described as follows:

a. *Primary porosity* is intergranular and controlled by deposition and lithification. It is highly interconnected and usually can be correlated with permeability since it is largely dependent on the geometry, size distribution and spatial distribution of the grains. The void systems of sands, sandstones and oolitic limestones are typical of this type.

b. *Secondary porosity* is foramenular and is controlled by fracturing, jointing and/or solution in circulating water although it may be modified by infilling as a result of precipitation. It is not highly interconnected and usually cannot be correlated with permeability. Solution channels or vugular voids developed during weathering or burial in sedimentary basins are indigenous to carbonate rocks such as limestones or dolomites. Joints or fissures which occur in massive, extensive formations composed of shale, siltstone, schist, limestone or dolomite are generally vertical, and they are ascribed to tensional failure during mechanical deformation (the permeability associated with this type of void system is often anisotropic). Shrinkage cracks are the result

[1] References given at end of paper.

of a chemical process (dolomitization) and do not appear to have any preferred orientation.

In the most general case, both classes of porosity are present and the internal-void volume of the rock is intermediate in nature, i.e., an independent system of secondary porosity is superimposed on the primary or intergranular system. The obvious idealization of an intermediate porous medium is a complex of discrete volumetric elements with primary porosity which are anisotropically coupled by secondary voids as shown in Fig. 1. This heterogeneous, double-porosity model will be investigated in detail since a significant number of petroleum reservoirs can be classified as intermediate. Although most intermediate-porosity rocks are limestones or dolomites, other sedimentary rocks such as cherty shale or siltstone exhibit this characteristic. Since the double-porosity model yields both of the single-porosity systems as limiting cases, an internal check on the results is provided.*

Sources of useful information for the characterization of a reservoir with intermediate porosity are limited and the information itself tends to be more qualitative than quantitative. The analysis of core data [9-11] is complicated by the presence of extraneous fractures induced during coring and retrieval, poor core recovery from intensely fractured zones, displacement of fracture surfaces and the size of the sample itself. In spite of these difficulties, acceptable values for the porosity and permeability of the intergranular material can be determined from plug-core data, and an estimate of the total porosity can be obtained from whole cores. The vertical extent, width, spacing, dip, relative strike, degree of infilling and the type of fracture can sometimes be qualitatively ascertained from a visual inspection of whole cores.

The volume and nature of the solution voids can be estimated from the examination of thin sections and/or polished surfaces. While it is usually not possible to correlate well logs on a well-to-well basis, an indication of the presence and vertical extent of fractured zones in an individual well may be obtained from sonic logs; similar qualitative in-

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*Analogous physical problems arise from the consideration of heat or mass transfer in a heterogeneous medium or electrical power transmission. Mathematically analogous problems occur when two flow processes are coupled by linear functions involving both dependent variables; e.g., chemical reaction, adsorption or phase equilibrium.
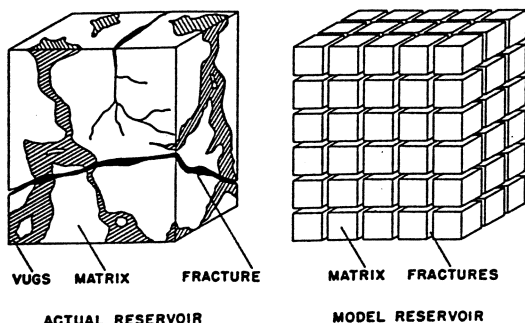

VUGS   MATRIX   FRACTURE   MATRIX   FRACTURES

ACTUAL RESERVOIR   MODEL RESERVOIR

FIG. 1 — IDEALIZATION OF THE HETEROGENEOUS POROUS MEDIUM.

formation may be obtained from spinner surveys, observed loss of circulation and down-hole photographs. It is clear that data obtained from these sources, which reflect conditions in the immediate vicinity of the well, give only a positive indication of the presence of secondary porosity and a description of the primary-porosity material. Some additional information can be acquired from non-routine laboratory tests on cores [3-5,12,13] if the scale of variation in the secondary porosity is sufficiently small.

Multi-well interference tests [14] or tracer tests can be utilized to establish the components of the anisotropic permeability and the orientation of the principal axes. The analysis of pressure build-up or fall-off data [15,16] permits the evaluation of the apparent permeability, (the geometric mean of the directional permeability components) the completion damage and, possibly, the primary and secondary pore volume.[17-19] This well-test information indicates the gross properties of the intermediate formation on an interwell scale.

To develop a plausible model for an intermediate reservoir, it is essential that all of the available measurements and observations, cited in the preceding paragraphs, be utilized; furthermore, the model must be consistent with the physical inferences obtained from the performance of actual reservoirs of this particular type. Applicable studies involving producing reservoirs located in many geographic areas are to be found in the literature. Reflecting these performance studies are a number of mechanistic models which have been proposed to describe the response of an intermediate reservoir to various natural and/or artificial drives. [20-29] Since our objective is to suggest a model which will simulate the behavior of a formation with intermediate porosity during single-phase flow, the more esoteric features of the proposed models can be discarded; the result is a physical idealization which incorporates a common set of characteristics which seem to be significant.

The model to be employed in this investigation is based on the following general assumptions:

a. The material containing the primary porosity is homogeneous and isotropic, and is contained within a systematic array of identical, rectangular parallelopipeds.

b. All of the secondary porosity is contained within an orthogonal system of continuous, uniform fractures which are oriented so that each fracture is parallel to one of the principal axes of permeability; the fractures normal to each of the principal axes are uniformly spaced and are of constant width; a different fracture spacing or a different width may exist along each of the axes to simulate the proper degree of anisotropy.

c. The complex of primary and secondary porosities is homogeneous albeit anisotropic; flow can occur between the primary and secondary porosities, but flow through the primary-porosity elements can not occur.

Additional assumptions of more particular nature will be made at appropriate points in the mathematical

treatment.

While the assumed model certainly implies heterogeneity on a macroscopic scale, it may be considered to be homogeneous if the dimensions of the homogeneous blocks are small in comparison with the dimensions of the reservoir, see the discussion by Warren, et al.[30] The same type of argument is often used to justify the measurement of porosity or permeability on a small core. The integration of the solution and fissure porosities is obviously necessary if the model is to be practically useful; furthermore, it is reasonable on the basis of similarity of flow performance. A certain amount of freedom has been allowed in the description of the fracture system to permit the use of all of the qualitative information that might be available.

Because of the discrete nature of the primary-porosity elements and evidence which indicates that normal and log-normal distributions of porosity and permeability are typical,[31] the arithmetic-mean porosity and the geometric-mean permeability should be used to obtain a "most probable" model.[32] The components of directional permeability and their orientation should be determined by means of interference tests. While this completes the description of the reservoir via current techniques, there are two parameters which are not yet determinate — the secondary porosity and a shape factor which describes the communication between the primary and secondary regions. An attempt will be made to develop methods of determining approximate values for these parameters from the known information and pressure data from well tests.

THEORY

Since it has been assumed that the reservoir can be treated as though it were homogeneous, let us define two pressures at each point (x, y, z, t) in the following manner:

$$p_1(x,y,z,t) = \int_V p(x,y,z,t)\, g_1(V)\, dV \Big/$$
$$\int_V g_1(V)\, dV \qquad \ldots \ldots (1)^*$$

$$p_2(x,y,z,t) = \int_V p(x,y,z,t)\, g_2(V)\, dV \Big/$$
$$\int_V g_2(V)\, dV \ldots \ldots (2)$$

where $g_1(V) = \begin{cases} 1, \text{in primary porosity} \\ 0, \text{outside of primary porosity} \end{cases}$

$g_2(V) = \begin{cases} 1, \text{in secondary porosity} \\ 0, \text{outside of secondary porosity} \end{cases}$

If useful solutions are to be obtained utilizing the "smoothed" pressures in the primary and secondary

regions, p_1 and p_2, (shown in Fig. 2) it is obvious that the volume considered must be small in comparison with the volume of the reservoir and must be large in comparison with the size of the matrix elements; i.e., the following conditions must be satisfied:

$$\int_V g_1(V)\, dV = \phi_1 \qquad \ldots \ldots \ldots (3)$$

$$\int_V g_2(V)\, dV = \phi_2 \qquad \ldots \ldots \ldots (4)$$

If the connate-water saturation in the secondary porosity is negligible, the average measured intergranular porosity ϕ_m is simply related to ϕ_1 and ϕ_2.

$$\phi_1 = (1 - \phi_2)\, \phi_m\, (1 - S_{wc})_m \qquad \ldots \ldots (5)$$

For a uniformly thick reservoir that is horizontal, homogeneous and anisotropic, the single-phase flow of a slightly compressible liquid is partially described by applying Green's theorem to the volume V to obtain the applicable form of the continuity equation.

$$\frac{k_{2x}}{\mu} \frac{\partial^2 p_2}{\partial x^2} + \frac{k_{2y}}{\mu} \frac{\partial^2 p_2}{\partial y^2} - \phi_1 C_1 \frac{\partial p_1}{\partial t} =$$
$$\phi_2 C_2 \frac{\partial p_2}{\partial t} \qquad \ldots \ldots \ldots \ldots \ldots (6)$$

where the x-axis and the y-axis coincide with the principal axes of permeability. In this expression C_1 and C_2 are total compressibilities; however, if it is assumed[27] that the external forces are constant, that there is no interaction between the two regions (ϕ_1 is independent of p_2 and ϕ_2 is independent of p_1) and that the variation of ϕ_2 with respect to p_2 is negligible, the following approximations obtain:

$$C_1 \cong c_o + \frac{c_p + S_{wc} c_w}{1 - S_{wc}} \qquad \ldots \ldots (7)$$

$$C_2 \cong c_o . \qquad \ldots \ldots \ldots \ldots (8)$$

For many of the pertinent cases, the second term in Eq. 7 will be at least as large as the compressibility of the flowing liquid.

In addition to Eq. 6, continuity on a local basis is necessary. If it is assumed that a quasi-steady state exists in the primary-porosity elements at all times, the following equation must be satisfied in the volume V surrounding each point in the reservoir:
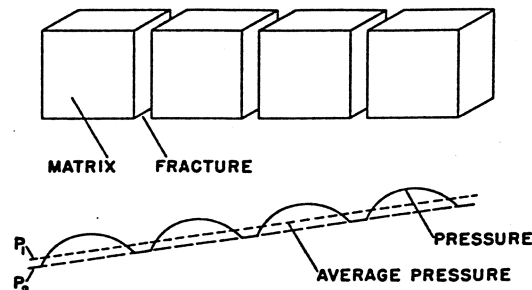

FIG. 2 — SCHEMATIC REPRESENTATION OF THE PRESSURE DISTRIBUTION IN THE MODEL.

$$\phi_1 C_1 \frac{\partial p_1}{\partial t} = \frac{\alpha k_1}{\mu}(p_2 - p_1) \quad \ldots \ldots (9)$$

The parameter α has the dimensions of reciprocal area; it is a shape factor which reflects the geometry of the matrix elements and it controls the flow between the two porous regions.* The assumption of a quasi-steady state will introduce some error into the solutions when times are small; but, since the results are ultimately to be used in conjunction with well-test data, the approximation should be adequate.

Eqs. 6 and 9 can be cast in dimensionless form by introducing appropriate transformations. For an infinite reservoir with a uniform initial pressure which is to be produced at a constant rate, the transformed equations, initial condition and boundary conditions can be written as follows:

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial \psi_2}{\partial \xi}\right) + \frac{1}{\xi^2}\frac{\partial^2 \psi_2}{\partial \theta^2} - (1-\omega)\frac{\partial \psi_1}{\partial \tau}$$

$$= \omega\frac{\partial \psi_2}{\partial \tau} \qquad \ldots \ldots \ldots (10)$$

$$(1-\omega)\frac{\partial \psi_1}{\partial \tau} = \lambda(\psi_2 - \psi_1) \quad \ldots \ldots (11)$$

$\psi_1 = \psi_2 = 0$; for $\tau = 0$, all ξ and θ

$\psi_2 = 0$; for $\tau > 0$, $\xi \longrightarrow \infty$, all θ

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{\partial \psi_2}{\partial n}\,d\theta = -1; \quad \tau > 0, \quad R =$$

$$= 1\Big/\sqrt{K\cos^2\theta + \frac{1}{K}\sin^2\theta} \quad \ldots (12)$$

where

$$\psi(\xi,\theta,\tau) = \frac{2\pi \bar{k}_2 h}{q\mu\beta}\Big[P - p(\xi,\theta,\tau)\Big]$$

$$\xi = \sqrt{\frac{x^2}{K r_w^2} + \frac{K y^2}{r_w^2}}$$

$$\theta = \tan^{-1}\left(\frac{Ky}{x}\right)$$

$$\tau = \bar{k}_2 t\big/(\phi_1 C_1 + \phi_2 C_2)\mu r_w^2$$

$$\bar{k}_2 = \sqrt{k_{2x} k_{2y}}, \quad K = \sqrt{k_{2x}/k_{2y}}$$

$$\lambda = \frac{\alpha k_1 r_w^2}{\bar{k}_2}$$

*For a more detailed discussion, see Appendix A.

$$\omega = \phi_2 C_2 \big/ (\phi_1 C_1 + \phi_2 C_2)$$

It is obvious that the well, which becomes an ellipse after transformation, causes the pressure distribution to lack axial symmetry. To avoid the necessity of including K as a parameter in the solution, it is assumed that the well-bore pressure obtained by solving (10–12) differs by a constant amount from the well-bore pressure obtained by solving the symmetrical problem ($K = 1$). After a very short time, the difference in these pressures due to the distortion of the pressure distribution in the region near the well will be constant;[2] therefore, since it has already been assumed that errors which are restricted to small times are permissible, the following expression completes the statement of the problem:

$$\psi_w(\tau) = \psi_2^*(1,\tau) - S^*. \quad \ldots \ldots (13)$$

where $\quad S^* = \ln\left(\frac{K+1}{2\sqrt{K}}\right)$

$\psi^{2*}(1,\tau) =$ solution of symmetrical problem, described by (10-12) with $K = 1$, evaluated at $\xi = 1$.

Using the Laplace transformation and solving the resulting equation subject to the boundary conditions, the desired solution can be written as follows:

$$\psi_2^*(1,\tau) = \mathcal{L}^{-1}\left[\frac{K_0(\sqrt{sf(s)})}{s\sqrt{sf(s)}\,K_1(\sqrt{sf(s)})}\right](14)$$

where $\quad f(s) = \dfrac{\omega(1-\omega)s + \lambda}{(1-\omega)s + \lambda}$

The asymptotic solution which is valid for $\tau > 100$** is obtained by making the usual substitutions, $K_0(\nu) \cong (-\gamma + \ln \nu - \ln 2)$ and $K_1(\nu) \cong 1/\nu$, and inverting.

$$\psi_2^*(1,\tau) \cong 1/2\left\{\ln \tau + .80908 + E_i\Big[-\lambda\tau/\omega(1-\omega)\Big]\right.$$
$$\left. - E_i\Big[-\lambda\tau/(1-\omega)\Big]\right\} \quad \ldots \ldots (15)$$

where $\quad -E_i(-v) = \displaystyle\int_v^\infty \frac{\exp(-u)\,du}{u}$

The solution for the case of a finite reservoir can be obtained in a similar manner if the condition at infinity in (12) is replaced by

$$\frac{\partial \psi_2}{\partial \xi} = 0; \quad \text{for } \tau > 0, \xi = R, \text{ all } \theta. \quad . (16)$$

If $R^2 \gg K$, Eq. 13 is valid for this case; and,

**This condition is sufficient for all values of λ and ω; however, if $\lambda \ll 1$, the condition is $\tau > 100\omega$; or, if $\omega \ll 1$, $\tau > (100\lambda - 1)/\lambda$.

$$\psi_2{}^*(1,\tau) \cong (\frac{2}{R^2-1})\left(\frac{1}{4} + \tau + \frac{(1-\omega)^2}{\lambda}\left\{1-\exp\right.\right.$$

$$\left.\left.\left[-\lambda\tau/\omega(1-\omega)\right]\right\}\right) - \left[\frac{3R^4-4R^4\ln R-2R^2-1}{4(R^2-1)^2}\right]$$

$$. . . . . (17)$$

DISCUSSION OF RESULTS

The solutions obtained emphasize the need for two parameters with which to describe a reservoir of the intermediate type; this agrees with the conclusions of Stewart, et al,[6] which were based on experimental work. The next step is to examine the effects of the parameters on reservoir behavior; then an attempt to evaluate them must be made.

If the derived results are to be accepted, it is necessary that the limiting behavior described by Eqs. 15 and 17 be physically reasonable. As $\omega \to 1.0$, both equations approach the asymptotic solution obtained by Van Everdingen and Hurst[33] for the same boundary conditions.* This behavior is proper since the primary porosity, or its effective compressibility, must vanish if ω is to approach unity; then, the reservoir contains only the homogeneous secondary porosity. Similarly, the equations indicate homogeneous behavior as $\lambda \to \infty$. This is also correct since there is no impedance to interporosity flow when λ approaches infinity, i.e., either k_1 or a must become infinite. Furthermore, the necessary condition that the infinite reservoir responds in a homogeneous manner for very large values of time is properly described.

Some results for the special case in which $\omega = 0$ (negligible storage capacity in the secondary porosity) in an infinite reservoir are shown in Fig. 3; this particular form of the model has been discussed quite extensively.[15,27] It is apparent that the dimensionless pressure drop $\psi_2{}^*(1,r)$ increases discontinuously when production begins, and it asymptotically approaches the homogeneous solution when λr becomes large. The pressure discontinuity is the result of a lack of fluid capacitance in the secondary porosity; the magnitude of the discontinuity is $\frac{1}{2}(-\gamma - \ln\lambda)$ for the cases of interest. Fig. 4 indicates the deviation from the asymptote for each case. The intercept at $r = 1$ is equal to the initial pressure discontinuity, and all of the linear segments have the same slope, i.e., -1.15/cycle. The apparent linearity of the curves persists until $\lambda r > 0.05$.

In Fig. 5, behavior patterns that are associated with finite values of ω are shown; once again the reservoir is infinite. The most notable feature is the second linear segment which is parallel to the asymptote but vertically displaced by an amount that is equal to $\ln(1/\sqrt{\omega})$. The transitional curve which connects the two linear portions represents the interaction between λ and ω. Deviations from the asymp-
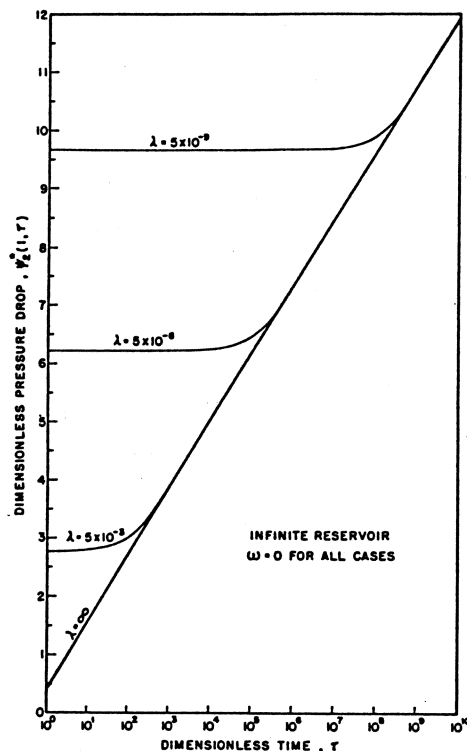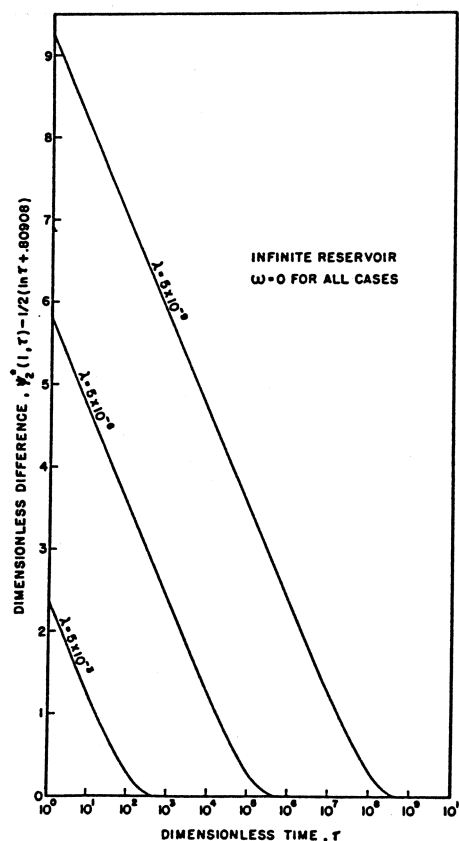
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*For $\nu \gg 1$, $-E_i$ $(-\nu) \cong \exp(\nu)/\nu$; and, for $\nu \ll 1$, $-E_i$ $(-\nu)$
$\cong - (\gamma + \ln\nu)$ where $\gamma = 0.5772 . . .$

FIG. 3 — SOLUTIONS OBTAINED BY APPROXIMATE INVERSION.

FIG. 4 — DEVIATION FROM THE ASYMPTOTE.

tote are presented in Fig. 6. The initial constant level is equal to the displacement of the second parallel segment from the homogeneous asymptote. The linear portion of the difference curve has a slope of -1.15/cycle and an intercept of $\frac{1}{2}[-\gamma-\ln\lambda+\ln(1-\omega)]$ at $\tau = 1$. Although the linear part of the deviation curve will not always be clearly defined, a line of the proper slope passing through the point of inflection will generally yield an intercept that will permit λ to be approximated quite adequately.

No results are presented for finite reservoirs since the behavior described by Eq. 17 is so familiar. The form of the equation is that which results in the classic time-lag curve [34] encountered commonly in the quasi-steady state. The asymptote for the dimensionless draw-down $\psi^*(1,r)$ is a linear function of r with a slope of $\frac{1}{2}(R^2-1)$ and an intercept approximately equal to $[\ln R - \frac{3}{4} + 2(1 - \omega)^2/\lambda R^2]$. The logarithm of the deviation from the asymptote is also a linear function of r; the slope is $[-\lambda/2.3\omega(1 - \omega)]$ cycles and the intercept at $r = 0$ is $\log[2(1 - \omega)^2/\lambda(R^2-1)]$. In principle, it is possible to evaluate R, λ and ω from the asymptote and the difference curve.

For practical use, equations, expressed in field units, for the draw-down and build-up histories and for the deviation of the actual build-up curve from its asymptote are listed in Appendix B. Theoretical build-up curves for various combinations of λ and ω are shown in Figs. 7 and 8 to illustrate the type of behavior that can be expected in an infinite reservoir. The assumed parameters and operating conditions used to generate the data are the following:

It is obvious that the behavior when there is only primary porosity $\omega = 0$ is so discontinuous that the build-up occurs almost instantaneously when λ is very small. The other cases for the smaller value of λ, 5×10^{-9}, give results that cause the reservoir to appear to have a closed boundary; furthermore, if only the early portion of the curve were recorded, the value of P, determined by extrapolation to $(\Delta t/t_s + \Delta t) = 1$, would be erroneously high by the quantity $m \log(1/\omega)$ and the value of the skin resistance S_d would be high by $1.15 \log (1/\omega)$.* Although

*The apparent value of the skin resistance in an anisotropic reservoir is actually equal to $S_d - S^*$; therefore, the correction should be made if well completions are to be evaluated on this basis. If a negative skin resistance is measured in a well that has not been stimulated, it may be used to obtain a lower limit on the anisotropy parameter K.

TABLE 1

$$\bar{k}_2 = 40 \text{ md}$$
$$h = 20 \text{ ft}$$
$$r_e = \infty$$
$$r_w = 0.316 \text{ ft}$$
$$\phi_1 C_1 + \phi_2 C_2 = 2.64 \times 10^{-6} \text{psi}^{-1}$$
$$\mu = 2 \text{ cp}$$
$$\beta = 1.23$$
$$S_d = S^*$$
$$P = 4000 \text{ psia}$$
$$q = 115 \text{ STB/D}$$
$$t_s = 21 \text{ days}$$

these particular results may be rather artificial, they do suggest the possibility of misinterpretation; e.g., the latter part of the transitional curve between the initial linear portion and the asymptote can be ana-
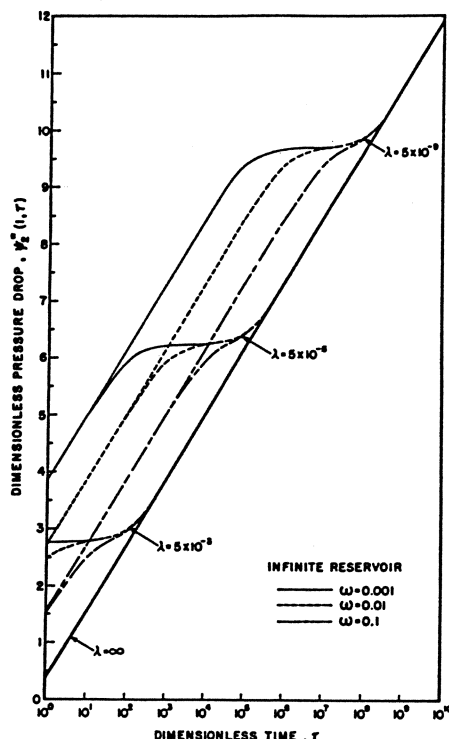
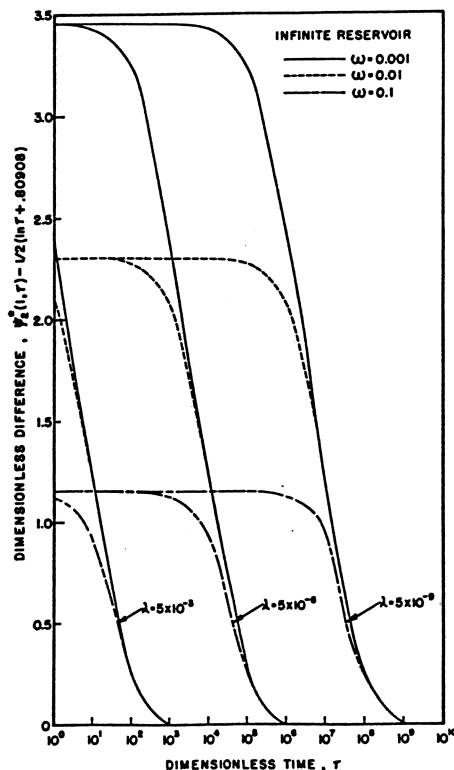

FIG. 5 — SOLUTIONS OBTAINED BY APPROXIMATE INVERSION.

FIG. 6 — DEVIATION FROM THE ASYMPTOTE.

lyzed by Pollard's method [17] or its extension [19] just as if the reservoir actually had a finite drainage radius.

For $\lambda = 5 \times 10^{-6}$, the curves, with the exception of $\omega = 0$, exhibit a double-slope type of build-up performance; apparently, this behavior is typical of fractured reservoirs.[16,17,35] Unfortunately, it is characteristic of stratified reservoirs as well, so the evaluation of such data may be uncertain. If, however, a plot of the deviation from the asymptotic solution behaves in the prescribed manner, it is safe to conclude that the reservoir can be considered to be intermediate in nature for predictive purposes. It may be postulated that evidence of this pressure lag indicates the presence of macroscopic heterogeneity or that λ is actually a direct measure of the scale of heterogeneity; e.g., for $\lambda \gg 1$, the reservoir behaves homogeneously.

When both of the parallel segments are discernible, the build-up analysis presents no problem. The later portion is interpreted in exactly the same manner as it would be if the reservoir were homogeneous. In addition to the possible errors which can result from using the wrong portion of the build-up curve, employing an incorrect method of analysis or assuming the wrong type of spatial permeability distribution, there is the possibility that a significant portion of the curve may be masked by afterflow effects. There is also a finite probability that the use of the "log $(P - p_w)$ vs time" method [16,17,19] could result in a complete description of a reservoir solely on the basis of after production. This difficulty can be circumvented by utilizing a recently developed tool which permits the well to be closed in at the formation.[36]

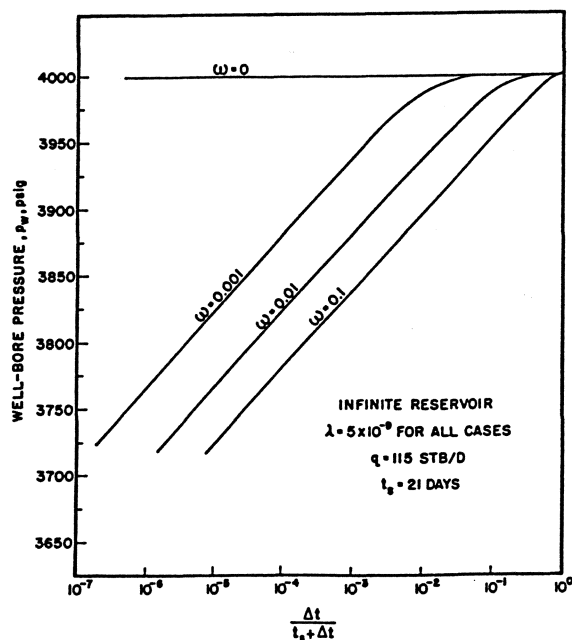No results are presented to demonstrate the expected behavior of a finite reservoir. This is done deliberately to avoid any chance of misunderstanding or misuse. Until the effect of the closed boundary is felt, the reservoir will act as though it were infinite, so the previously discussed methods can be applied. Beyond that time, the difficulties inherent in the procurement of sufficient, accurate data and the fundamental uncertainties in the available methods of analysis force us to question the value of such tests. The time required to achieve quasi-steady state conditions in a heterogeneous reservoir is one or two orders of magnitude greater than it is in a homogeneous reservoir.[35] The accurate determination of slowly varying pressures is quite difficult unless a pressure-sensing device that is operating in the correct range is being used (it might be necessary to change pressure bombs several times to accomplish this). Finally, the analysis of these small changes in pressure may become largely a matter of personal judgment; and, if higher order pressure differences are involved, the results may be more artistic than scientific. To emphasize these points, the dimensionless pressure history of a single, undamaged well producing at a constant rate from an infinite homogeneous reservoir [33] was used together with the reservoir data in Table 1, $q = 200$ STB/D and $t_s = 10$ days, to obtain the synthetic build-up curve shown in Fig. 9. The data are plotted in the prescribed manner [17,19] and the graphical analysis is shown. The interpretation of the parameters so determined leads to the following results:

pore volume of the coarse voids = 1,100,000 cu ft

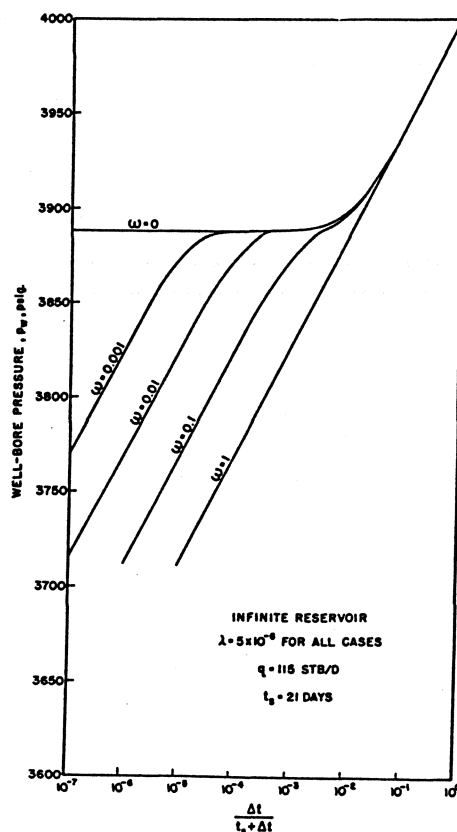FIG. 7 — THEORETICAL BUILD-UP CURVES.

FIG. 8 — THEORETICAL BUILD-UP CURVES.

pore volume of the fine voids = 5,670,000 cu ft
radius of the reservoir = 904 ft
radius of the damaged region = 130 ft

It is obvious that this approach yields erroneous results. The fault cannot be assigned entirely to the model upon which the technique is based since the model is physically reasonable during quasi-steady state flow. Rather, the difficulty lies in the nature of the data used. It is probable that any decaying function can be approximated by a series of exponential terms over an interval of gradual change; thus, ambiguity results.

As a consequence of the examination of the approximate mathematical treatment of the idealized physical model, it has been suggested that the two additional parameters which are significant in the behavior of an intermediate reservoir can be evaluated from pressure draw-down or build-up data. The data should be obtained before the effect of the reservoir boundary is felt at the well. Extreme caution should be used in the analysis of data secured during the later stages of build-up in a finite reservoir.

SUMMARY AND CONCLUSIONS

An investigation has been conducted for the purpose of improving the description of a formation which contains both primary (intergranular) porosity and secondary (fissure and/or vugular) porosity. Based on the assumption that the primary-porosity region contributes significantly to the pore volume but contributes negligibly to the flow capacity, an idealized model has been developed to study the characteristic behavior of such a reservoir. Unsteady-state flow in this model reservoir has been described mathematically, and asymptotic solutions have been obtained. The pressure build-up performance has been examined in some detail; and, a technique for analyzing the build-up data to evaluate the desired parameters has been suggested. The use of this approach in the interpretation of field data has been discussed.

As a result of this study the following general conclusions can be drawn:

1. Two parameters are sufficient to characterize the deviation of the behavior of a medium with "double porosity" from that of a homogenous, porous medium. One of the parameters, ω, is a measure of the fluid capacitance of the secondary porosity and the other, λ, is related to the scale of heterogeneity that is present in the system.
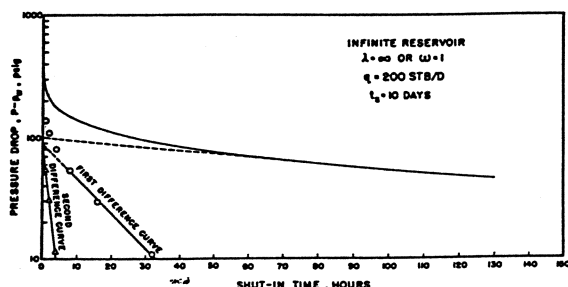
2. These parameters can be evaluated by the proper analysis of pressure build-up data obtained from adequately designed tests.

3. Since the build-up curve associated with this type of porous medium is similar to that obtained from a stratified reservoir, an unambigous interpretation is generally not possible without additional information.

4. Methods which utilize differences obtained from pressure data recorded during the final stages of a build-up test should be used with extreme caution.

NOMENCLATURE

C = total compressibility, LT^2/M

$K = \sqrt{k_{2_x}/k_{2_y}}$, degree of anisotropy, dimensionless

P = initial pressure, M/LT^2

$R = r_e/r_w$, radius of closed boundary, dimensionless

$S^* = \ln[(k+1)/2\sqrt{K}]$, effective skin resistance due to anisotropy, dimensionless

S_d = skin resistance due to completion, dimensionless

S_{wc} = connate water saturation, dimensionless

c_o = compressibility of flowing liquid, LT^2/M

c_p = effective pore compressibility, LT^2/M

c_w = compressibility of connate water, LT^2/M

b = thickness of formation, L

k = absolute permeability, L^2

$\bar{k}_2 = \sqrt{k_{2_x} k_{2_y}}$, effective permeability of anisotropic medium, L^2

ℓ = characteristic dimension of heterogeneous region, L

p = pressure, M/LT^2

p_w = well-bore pressure, M/LT^2

Δp_w = deviation of actual pressure from the value on its asymptote, M/LT^2

q = production rate, L^3/T

s = Laplace operator, $1/T$

t = time, T

t_s = time of shut-in, T

$\Delta t = t - t_s$, time elapsed since shut-in, T

x, y, z = rectangular coordinates; the axes coincide with the principal axes of permeability, L

a = geometric parameter for heterogeneous region, $1/L^2$

β = formation volume factor, dimensionless

γ = 0.5772. . ., Euler-Mascheroni constant, dimensionless

$\lambda = ak_1 r_w^2/k_2$, the parameter governing interporosity flow, dimensionless

μ = viscosity, M/LT

$\xi = \sqrt{\dfrac{x^2}{Kr_w^2} + \dfrac{Ky^2}{r_w^2}}$, radial coordinate, dimensionless

FIG. 9 — WORK CURVE FOR BUILD-UP ANALYSIS (REFERENCES 19, 21).

$r = \bar{k}_2 t / (\phi_1 C_1 + \phi_2 C_2)\mu r_w{}^2$, time, dimensionless

r_s = time of shut-in, dimensionless

$\Delta r = r - r_s$, time elapsed since shut-in, dimensionless

ϕ = porosity, dimensionless

$\psi = 2\pi\bar{k}_2 h \ (P-p)/q\mu\beta$, pressure decline, dimensionless

$\psi_2{}^*$ = pressure decline in secondary porosity at r_w, dimensionless

$\psi_w = \psi_2{}^* - S^*$, pressure decline at well, dimensionless

$\omega = \phi_2 C_2 / (\phi_1 C_1 + \phi_2 C_2)$, parameter relating fluid capacitance of the secondary porosity to that of the combined system, dimensionless

SUBSCRIPTS

1 = primary porosity

2 = secondary porosity

m = matrix (refers to properties measured on small core samples)

x, y, z = vector component

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APPENDIX A

ADDITIONAL THEORETICAL CONSIDERATIONS

Prior to the derivation of Eqs. 6 and 9, it was assumed that the secondary porosity was contained within an array of homogeneous, geometrically identical elements. Since it is ultimately necessary to introduce average values in order to obtain useful results, this simplification is justified. To show that there is no loss of generality, an alternate approach will be considered.

Let us define $p_1 \ (x, y, z, t)$ within a representative volume in the following manner:

$$p_1(x,y,z,t) = \frac{1}{\phi_1} \int_0^\infty v(\ell) \phi_1{}'(\ell) \bar{p}_1(\ell;t) \, d\ell$$

$$\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (1\text{-}1)$$

where $\phi_1 = \int_0^\infty v(\ell) \phi_1{}'(\ell) \, d\ell$

$$\phi_1{}'(\ell) = \phi_m(\ell) \left[1 - S_{wc}(\ell) \right]_m$$

$$\bar{p}_1(\ell;t) = \frac{1}{V(\ell)} \int_0^\ell p(\ell; v, t) A(v) \, dv$$

$v(\ell)\,d\ell$ = bulk volume of matrix material with characteristic dimension from ℓ to $\ell + d\ell$ per unit bulk volume;

$$\int_0^\infty v(\ell)\,d\ell = 1 - \phi_2$$

$$V(\ell) = \int_0^\ell A(v)\,dv = \text{charcteristic volume associated with the dimension } \ell.$$

The interporosity flow per unit volume, q, is given by

$$q = \frac{1}{\mu \phi_1}\left[\int_0^\infty v(\ell)\phi_1'(\ell)k_2(\ell)a(\ell)(p_2 - p_1)\,d\ell\right]$$

$$\dotfill (1\text{-}2)$$

where $p_2(x,y,z,t)$ is assumed to be only a function of time over the region of interest. Let

$$ak_1 = \frac{\int_0^\infty v(\ell)\phi_1'(\ell)k_1(\ell)a(\ell)(p_2 - \bar{p}_1)\,d\ell}{\int_0^\infty v(\ell)\phi_1'(\ell)(p_2 - \bar{p}_1)\,d\ell}$$

$$\dotfill (1\text{-}3)$$

Substituting (1-3) in (1-2),

$$q = \frac{ak_1}{\mu \phi_1}\left[p_2\int_0^\infty v(\ell)\phi_1'(\ell)\,d\ell - \int_0^\infty v(\ell)\phi_1'(\ell)\bar{p}_1\,d\ell\right]$$

$$\dotfill (1\text{-}4)$$

Utilizing previously defined quantities,

$$q = \frac{ak_1}{\mu}(p_2 - p_1). \dotfill (1\text{-}5)$$

Similarly,

$$q = \phi_1 C_1 \frac{\partial p_1}{\partial t} = \int_0^\infty v(\ell)\phi_1'(\ell)C_1(\ell)\frac{d\bar{p}_1}{dt}\,d\ell \cdot (1\text{-}6)$$

where $C_1 = \dfrac{\int_0^\infty v(\ell)\phi_1'(\ell)C_1(\ell)\dfrac{d\bar{p}_1}{dt}\,d\ell}{\int_0^\infty v(\ell)\phi_1'(\ell)\dfrac{d\bar{p}_1}{dt}\,d\ell}$

Therefore, quite generally,

$$\phi_1 C_1 \frac{\partial p_1}{\partial t} = \frac{ak_1}{\mu}(p_2 - p_1) \dotfill (1\text{-}7)$$

It is apparent that the characteristic dimension ℓ must be examined more closely. In principle, the three dimensions of the identical matrix blocks and the width of the fracture (or one dimension and the three fracture widths) can be calculated if k_{2_x}, k_{2_y}, and k_{2_z} and ϕ_2 are known. Since this information will not generally be available, it is necessary to resort to approximations. The simplest approach is to assume uniformly spaced fractures and to allow variations in the fracture width to satisfy the conditions of anisotropy. Then,

$$a = 4n(n+2)/\ell^2 \dotfill (1\text{-}8)*$$

where n = number of normal sets of fractures = 1,2,3. Let us define ℓ in terms of measurable parameters.

$$\ell = 2\left[n(n+2)\right]^{1/2}\left(\frac{k_1 r_w^2}{\lambda k_2}\right)^{1/2} \dotfill (1\text{-}9)$$

The following min-max approximations obtain:

$$\ell = 4.78\left(\frac{k_1 r_w^2}{\lambda \bar{k}_2}\right)^{1/2} ; \quad n = 1,2,3. \quad (1\text{-}10)$$

where the maximum error is 38 per cent.

$$\ell = 4.30\left(\frac{k_1 r_w^2}{\lambda \bar{k}_2}\right)^{1/2} ; \quad n = 1,2. \quad (1\text{-}11)**$$

where the maximum error is = 24 per cent.

$$\ell = 6.55\left(\frac{k_1 r_w^2}{\lambda \bar{k}_2}\right)^{1/2} ; \quad n = 2,3. \quad (1\text{-}12)$$

where the maximum error is = 16 per cent.

If the blocks actually have the dimensions $x = a$, $y = b$ and $z = c$, the equivalent value of ℓ can be estimated from the surface-volume ratio; i.e.,

$$\ell = \frac{3abc}{(ab + bc + ca)} ; \quad n = 3 \dotfill (1\text{-}13)$$

$$\ell = \frac{2ab}{(a+b)} ; \quad n = 2 \dotfill (1\text{-}14)$$

$$\ell = a ; \quad n = 1. \dotfill (1\text{-}15)$$

APPENDIX B

BEHAVIOR EQUATIONS

In the equations that follow, these definitions obtain:***

$$\tau = (2.637 \times 10^{-4})\,\bar{k}_2 t /(\phi_1 C_1 + \phi_2 C_2)\,\mu r_w^2$$

---

*Indicative of the insensitivity of a to the exact geometry is the fact that the results for an n-dimensional sphere agree with (1-8) if ℓ is replaced 2r.

**This is the most probable case since it is unlikely that horizontal fractures contribute significantly to the flow capacity.

***Units are psia, STB/D, res. bbl, md, ft, psi^{-1} and hours.

$$R = r_e / r_w$$

$$m = 162.5 \, q\mu\beta / \bar{k}_2 h$$

$$S^* = \ln\left(\frac{K+1}{2\sqrt{K}}\right)$$

S_d = skin resistance due to completion

$\Delta p_w(t)$ = difference between the asymptote and the actual curve

INFINITE RESERVOIR

a) Draw-Down, $\tau > 100\,\omega$ if $\lambda << 1$ or
$$\tau > 100 - \frac{1}{\lambda} \text{ if } \omega << 1$$
or $\tau > 100$ (all cases)

$$p_w(\tau) \cong P - m\left\{\begin{array}{l} \log \tau + .351 + .435 E_j\left[-\lambda\tau/\omega(1-\omega)\right] \\ -.435 E_j\left[-\lambda\tau/(1-\omega)\right] - .87 S^* + .87 S_d \end{array}\right\}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots \text{(II-1)}$$

b) Build-Up, $\lambda\tau_s > 3$

$$\Delta\tau > 100\omega \text{ if } \lambda << 1 \text{ or}$$
$$\Delta\tau > 100 - \frac{1}{\lambda} \text{ if } \omega << 1$$

$$p_w(\tau_s + \Delta\tau) \cong P - m\left\{\log\frac{\tau_s + \Delta\tau}{\Delta\tau} - .435 E_j\left[-\lambda\Delta\tau/\right.\right.$$
$$\left.\left. \omega(1-\omega)\right] + .435 E_j\left[-\lambda\Delta\tau/(1-\omega)\right]\right\} \quad \cdots \text{ (II-2)}$$

$$\Delta p_w(\tau_s + \Delta\tau) \cong .435 m\left\{E_j\left[-\lambda\Delta\tau/\omega(1-\omega)\right] - E_j\right.$$
$$\left.\left[-\lambda\Delta\tau/(1-\omega)\right]\right\};$$

all values of ω and all values of $\Delta\tau$ (II-3)

$$\Delta p_w(\tau_s + \Delta\tau) \cong -m\log\omega; \, \omega \neq 0 \text{ and small}$$
values of $\Delta\tau$ $\cdots\cdots\cdots\cdots$ (II-4)

(constant portion of curve)

$$\Delta p_w(\tau_s + \Delta\tau) \cong m\left[-.251 - \log\lambda + \log(1-\omega) - \log\Delta\tau\right];$$

all values of ω and intermediate values of
$\Delta\tau$ $\cdots\cdots\cdots\cdots\cdots$ (II-5)

(linear portion of curve)

FINITE RESERVOIR

a) Draw-Down, $\tau > 100\omega R^2$ if $\lambda << 1$ or $\tau > 100 R^2 - \frac{1}{\lambda}$

$$p_w(\tau) \cong P - \frac{1.74m}{R^2}\left(\tau + \frac{(1-\omega)^2}{\lambda}\left\{1 - \exp\left[-\lambda\tau/\omega(1-\omega)\right]\right\}\right) -$$
$$-.87m\left(\ln R - .75 - S^* + S_d\right) \cdots\cdots \text{ (II-6)}$$

b) Build-Up, $\lambda\tau_s > 5\omega$

$$\Delta\tau > 100\omega R^2 \text{ if } \lambda << 1$$
$$\Delta\tau > 100 \, R^2 - \frac{1}{\lambda} \text{ if } \omega << 1$$

$$p_w(\tau_s + \Delta\tau) \cong P - \frac{1.74m}{R^2}\left(\tau_s + \frac{(1-\omega)^2}{\lambda}\exp\left[-\lambda\Delta\tau/\omega(1-\omega)\right]\right)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots \text{(II-7)}$$

$$\log\left[\Delta p_w(\tau_s + \Delta\tau)\right] = \log\left[\frac{1.74m(1-\omega)^2}{\lambda R^2}\right] -$$
$$-.435\left[\lambda\Delta\tau/\omega(1-\omega)\right]\cdots\cdots\cdots \text{ (II-8)}$$

***